AD-A045 827 COLORADO UNIV BOULDER DEPT OF ELECTRICAL ENGINEERING F/G 9/4
A SYNTHESIS THEORY FOR MULTIPLE-LOOP OSCILLATING ADAPTIVE SYSTE--ETC(U)
1977 I HOROWITZ, A SHAPIRO AFOSR-76-2946
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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: AFOSR-TR-77-1221	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): A SYNTHESIS THEORY FOR MULTIPLE-LOOP OSCILLATING ADAPTIVE SYSTEMS	5. TYPE OF REPORT & PERIOD COVERED: Interim rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Isaac Horowitz, Aharon Shapiro	8. CONTRACT OR GRANT NUMBER(s): AFOSR-76-2946
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of Colorado, Department of Electrical Engineering, Boulder, Colorado 80309	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, 2304/A1
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB DC 20332	12. REPORT DATE: 1977
	13. NUMBER OF PAGES: 35
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 39p.	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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20. ABSTRACT (Continued)

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high-frequency gain uncertainty $\rho = k_{max}/k_{min}$, the factor which is generally responsible for large 'cost of feedback'. It is however necessary to design these systems such that the respons is essentially quasilinear to the desired class of command and disturbance signals. In the SOAS, ρ (RHO) reappears as a significant factor in the quasilinear requirements which may, depending on the numbers involved, completely vitiate its banishment as an uncertainty factor. The development of a quantitative design theory for the SOAS pinpoints the two-loop SOAL extension needed to overcome this SOAS limitation, and the development of a similar SOAL quantitative design theory. In the latter, ρ (RHO) disappears from both the adaptive and quasilinear conditions, but reappears as a factor in the rate of adaptation of the system. It may be banished from here too, by means of a three-loop self-oscillating system (SOANL), for which the SOAL design theory is applicable with minor extensions.

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A SYNTHESIS THEORY FOR MULTIPLE-LOOP OSCILLATING ADAPTIVE SYSTEMS

Isaac Horowitz† and Aharon Shapiro†

ABSTRACT

The multiple-loop self-oscillating adaptive (SOAL) system is presented as a natural, logical means of overcoming a serious limitation of the single-loop self-oscillating system (SOAS). Both structures have the property of zero sensitivity to plant high-frequency gain uncertainty $\rho = k_{max}/k_{min}$, the factor which is generally responsible for large 'cost of feedback'. It is however necessary to design these systems such that the response is essentially quasilinear to the desired class of command and disturbance signals. In the SOAS, ρ reappears as a significant factor in the quasilinear requirements which may, depending on the numbers involved, completely vitiate its banishment as an uncertainty factor. The development of a <u>quantitative</u> design theory for the SOAS pinpoints the two-loop SOAL extension needed

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* This research was supported in part by the U.S. Air Force Office of Scientfic Research under Grant No. AFOSR-76-2946 at the University of Colorado.

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A SYNTHESIS THEORY FOR MULTIPLE-LOOP OSCILLATING ADAPTIVE SYSTEMS

I. INTRODUCTION

It is well known that high frequency dither can linearize the response of a certain class of nonlinear elements to slower and smaller amplitude signals [1-3]. This phenomenon is explicable by multiple-input describing function theory [4-7], and has recently received more abstract and rigorous treatment [8]. The adaptive property of such dither was later recognized [9, 10 and especially 11]. Structures similar to dithered systems were among those promoted in the USAF sponsored competition for adaptive flight control systems [12, 13]. Some versions were among the successful ones, used for example in the X-15, X-20 involving fantastic parameter uncertainty factors of more than one thousand [14-16]. These systems contain strongly nonlinear elements but their system properties have significant linear aspects. Thus, aside from their inherent value, they are of interest as a transition between the linear and nonlinear adaptation philosophies.

A variety of dithered feedback systems exist. In most, the amplitude of the dither is used as a parameter identifier, but in some the phase is also used. Single and two-loop, self-oscillating and externally excited systems have been described [17-22]. In all, both the quasilinear and the adaptive aspects are essential features which must be properly designed into the system. The quasilinear aspect has received thorough quantitative treatment [24], but the adaptive aspect to a much lesser extent. Since parameter uncertainty is the motivation for these systems, surely the actual extent of uncertainty and the desired system response tolerances, ought to appear as significant design parameters. It is reasonable to expect that an optimum design to handle an uncertainty factor of say 1000, should be significantly different from one for an uncertainty factor of 100. But this is not so in most of the literature. There have so far appeared quantitative design theories only for the single-loop self-oscillating (SOAS) and the single-loop externally excited (EEAS) adaptive systems [25, 26]. This paper presents such a design theory for two versions of the two-loop self-oscillating system (SOAL and SOANL). The SOAL structure used here is not new, although it was independently derived in a straightforward logical manner (as will be seen), in the process of trying to overcome SOAS limitations. The quantitative design theory is new and reveals the the superiority of the SOAL structure over other two-loop versions. The quasilinear feastures common to all dithered systems are first reviewed.

<u>Review of Quasilinear Theory</u> [24]

Let the input to the nonlinearity N (Fig. 1) be

$$x = A \sin \omega_o t + x_f(t) \triangleq x_o(t) + x_f(t) \qquad (1a,b)$$

Let ω_f be the bandwidth of $X_f(j\omega) = \mathcal{L}[x_f(t)]$, $s = j\omega$ and α, β specific suitable numbers ($\alpha = \beta = 3$ will be used in subsequent numerical examples). For a certain class of nonlinearities, if

$$\sup_t |x_f(t)| \leq \frac{A}{\alpha}, \; \omega_f \leq \frac{\omega_o}{\beta} \qquad (2a,b)$$

then y, the output of N may be written with good accuracy (depending on the values of α, β [24])

$$y = N_o A \sin \omega_o t + N_f x_f(t); \quad N_o = \frac{M}{A}, \; N_f = \frac{N_o}{2} \qquad (3a\text{-}c)$$

with M a real parameter of N. The ideal relay with $\pm B$ saturating levels will be always used henceforth for illustrative purposes, for which $M = \frac{4B}{\pi}$. Thus, N presents the gain N_o to the faster and larger oscillating component x_o and $N_f = N_o/2$ to the slower and smaller forced component x_f of (1a,b). Accordingly in Fig. 1 the forced component system response functions to command and disturbance inputs are

$$T_f(s) \triangleq \frac{C_f}{R} = G_3 \frac{L_f}{1+L_f}, \; T_d \triangleq \frac{C_f}{D} = \frac{1}{1+L_f}, \qquad (4a,b,c)$$

$$T_{dz} \triangleq \frac{Z_d}{D} = \frac{-L_f}{1+L_f}.$$

with forced and oscillating component loop transfer functions

$$L_f = G_1 G_2 N_f P = G_1 G_2 N_f K P_h, \quad L_o = G_1 G_2 N_o P = 2L_f,$$

$$N_f = \frac{N_o}{2} = \frac{2B}{\pi A}. \qquad (5a,b,c)$$

In the above $P = K P_h$ is the constrained plant and it is assumed in the meantime that the only uncertainty is in the high frequency plant gain factor K, defined by

$$\lim_{s\to\infty} \frac{P(s)}{K} \stackrel{\Delta}{=} \frac{1}{s^p} \tag{6}$$

where p is the excess of plant poles over zeros.

II. REVIEW OF SOAS ADAPTIVE THEORY

There is a logical and natural transition from the SOAS to the two-loop SOAL, which renders the latter theory quite transparent once the former is well understood. First, note that if the excess of poles over zeros of $L_o(s)$ of (5b) is ≥ 3, then from describing function theory and the infinite gain of the ideal relay (or large enough gain for other nonlinear element), there is a limit cycle at ω_o, defined by

$$\text{Arg } L_o(j\omega_o) = -\pi \quad \text{with} \quad |L_o(j\omega_o)| = \frac{4B}{\pi A}|G_1 G_2 \; KP_h(j\omega_o)| = 1$$

(7a, b)

Since all elements of (7b) except K, A are fixed, K/A must be a constant, giving zero sensitivity of $L_f(j\omega) = L_o/2$ to K uncertainty. Incidentally, the rigorous requirements for excellent accuracy of the describing function [27] are easily achieved in SOAS and SOAL design. Second, in any practical system there must be a bound m on the tolerable amplitude of the limit cycle at the system output, i.e. in Fig. 1

$$\sup_{K} \frac{4B}{\pi} |G_2\ K\ P_h\ (j\omega_o)| = \frac{4B}{\pi} |G_2\ K_{max}\ P_h(j\omega_o)| \leq m \qquad (8)$$

Third, the quasilinearity conditions (2a,b) compel specification of the extreme command and disturbance signals for which quasilinearity is to be valid. It is found [25] that these are the inputs giving maximum forced component plant output $|Z_e(j\omega_o)|$ at ω_o in Fig. 1, and denoted by $Z_{er} = R_e\ T_f$ of (4) for the extreme command input, and $Z_{ed} = -D_e\ T_{dz}$ of (4) for the extreme disturbance input. The corresponding extreme input to N is denoted by X_{ei} with $Z_{ei}\ (s) = \frac{2B}{\pi A}\ G_2\ KP_h\ X_{ei}(s)$ in Fig. 1. If $d(t)$, $r(t)$ never or rarely occur simultaneously then the larger at each ω must be used. If the two extremes do occur together, then their sum must be used. Fourth, a good optimization criterion is the minimization of ω_o of (7), because the bandwidth of L_f is thereby minimized [28].

In the SOAS, conditions (2a,8) usually determine $(\omega_o)_{min}$, as follows. Write $Z_e(j\omega) = \frac{2B}{\pi A} G_2KP_hX_e(j\omega) = \frac{2B}{\pi A_{min}} G_2\ K_{min}\ P_h\ X_e(j\omega)$ because A/K is invariant, as previously noted following (7). Use the above to eliminate BG_2P_h in (8), giving

$$\left|\frac{X_e}{A_{min}}\ (j\omega_o)\right| \geq \frac{2\ K_{max}}{m\ K_{min}}\ |Z_e(j\omega_o)| \stackrel{\Delta}{=} \gamma_1 |Z_e(j\omega_o)| \qquad (9)$$

The final important constraint is (2a) in which the worst case is when $x_f = x_e$ and $A = A_{min}$. It is shown in [25] that the best that can be done results in (2a) being equivalent to

$$\left| \frac{X_e}{A_{min}} (j\omega) \right| \leq \frac{1}{\alpha\,\omega}, \qquad \omega \leq \omega_o . \tag{10}$$

The importance of conditions (9, 10) is illustrated by the following desing example.

<u>Design Example.</u> $P = \frac{K}{s}$, $K \in [1,100]$. The extreme command input $R_e(s) = \frac{75}{s}$ and the disturbancesare thought to be negligibly small. The maximum tolerable oscillation amplitude $m = 1$. The desired system response function is

$T_f(s) = \frac{0.01\,\Phi(s)}{s^2+0.1s+0.01}$, where $\Phi(s)$ contains the far-off

poles and zeros which may be added, if worthwhile.

<u>Design.</u> In the above $\gamma_1 = 2\,K_{max}/m\,K_{min} = 200$, $Z_e = Z_{re} = R_e\,T_f$ and the resulting $\gamma_1|Z_{er}(j\omega)|$ is sketched in Fig. 2 for $\Phi(s) = 1$ as is $1/\alpha\,\omega$ $(\alpha = 3)$ of (10), as if testing any ω if it may be used for ω_o. Conditions (9,10) dictate that $(\omega_o)_{min}$ is at the inter-action of the two curves, with value 20 rps. Suppose $\Phi(s) = \frac{1}{(s+1)^2}$ is acceptable. The new $\alpha_1|Z_e(j\omega)|$ gives $\omega_{o_{min}} = 4.5$. Clearly, it is desirable to add as many far-off poles as tolerable. Condition (2b) is then satisfied

by a large margin.

It is seen that $\gamma_1 = \frac{2K_{max}}{mK_{max}}$ is decisively important in determining $(\omega_o)_{min}$. It is interesting that the uncertainty in K , expressed by $\frac{K_{max}}{K_{min}}$, which was banished from L_f , returns as a constraint which is very important in determining $(\omega_o)_{min}$ and with it, the bandwidth of L_f . If γ_1 is large enough, the resulting L_f bandwidth may be larger than that required by a linear time-invariant feedback design satisfying the same quantitative specifications [28].

Limitations due to Disturbances

Disturbances impose a serious limitation on the SOAS. This is seen by asking: Given ω_o determined by Z_{er} as in the above example, for what class of disturbances D in Fig. 1, is quasilinearity maintained? Consider then $Z_e(j\omega_o) = Z_{de}(j\omega_o) = \frac{-L_f}{1+L_f} D_e(j\omega_o) = D_e(j\omega_o)$, because from (5b, 7b) $L_f(j\omega_o) = -0.5$. In the limiting case (from 9, 10) $\gamma_1|Z_{de}(j\omega_o)| = \gamma_1|D_e(j\omega_o)| = 1/3\omega_o$, which determines the tolerable D class. Thus in the above example if $D_e = k/s$, then set $200\, k_{max}/\omega_o = 1/3\, \omega_o$, giving $k_{max} = 1/600$ - a very small tolerable step disturbance - the assumed negligible disturbance may not be ignorable after all. If $D_e = \frac{k}{(s+\alpha)^2}$ with $\omega < \omega_o$, then

$200 \frac{k_{max}}{\omega_o^2} \doteq \frac{1}{3\omega_o}$ gives $k_{max} = \frac{\omega_o}{600}$. Any larger disturbances violate quasilinearity condition and the response to them becomes very sensitive to the relative phases of the sinusoid and the disturbance input.

In the above, the problem of design for specific disturbance attenuation was not even considered. Suppose there are significant disturbances which must be attenuated. Whatever disturbance attenuation design philosophy [28] is used, it results in some extreme plant output Z_{de} for the extreme disturbance input D_e . As in the above, it is necessary that $\gamma_1 |D_e(j\omega_o)| \leq 1/\alpha\omega_o$. If $D_e(s) = k/s$, there is no solution if $\gamma_1 k > 1/\alpha$. However if the excess of poles over zeros of $D_e(s) \geq 2$, then there is always a solution, but $(\omega_o)_{min}$ may then be exceedingly large. Thus, in the above example, $D_e = J/s^2$ gives $\gamma_1 J/\omega_o^2 = 1/3\omega_o$, so $(\omega_o)_{min} = 6000\ J$. A linear time-invariant design might be far better in terms of loop-bandwith.

<u>Completion of Design Example</u>

If disturbance attenuation is of no concern, then the design is basically complete, once ω_o has been chosen [25]. For $\omega < \omega_o$, $|L_o(j\omega)|$ should be chosen >1 of course, but not necessarily very large, because L_f as such is not needed for disturbance attenuation or sensitivity reduction. It is desirable to have large $|d \text{ Arg } L_o(j\omega)/d\omega|$ at ω_o , so that the value of ω_o is insensitive to changes in plant dynamics. Such an Arg $L_o(j\omega)$ is associated with fast decrease of $|L_o(j\omega)|$ near ω_o, which in itself is also desirable. If disturbance attenuation is of concern then $L_f = 0.5\ L_o$ must be shaped to obtain the desired disturbance attenuation, which must, as previously shown, be compatible with the chosen ω_o. In fact, from the previous discussion, $\omega_{o_{min}}$ due to quasilinearity constraints is very likely much larger than that needed for disturbance attenuation alone. As an example of design for (compatible) disturbance attenuation suppose one wants it (i.e, $|T_d(j\omega)|$ of 4b) to be $\leq B_d(\omega)$, a straight line on a Bode plot (db vs log ω), with $B_d(1) = -5$db, $B_d(\cdot 1) = -30$db. It is given that this bound applies for $\omega < 1.2$. For all other ω it is required that

$\left|\frac{L_f}{1+L_f}\right| \leq 2.3$db. See [28, 29 Sec. 10. 15] how such bounds may arise. In any case, no matter what design approach is used, disturbance attenuation is a matter of small enough $|T_d(j\omega)|$ vs ω over a sufficiently large ω range. Different techniques only give different $B_d(\omega)$ for $T_d(j\omega)$. Since $T_d = \frac{1}{1+L_f}$, the $B_d(\omega)$ generate bounds $B(\omega)$ on $L_f(j\omega)$, shown in Fig. 3, which together with the previous constraint of $\omega_{o\ min} = 4.5$ and $L_f(j\omega_o) = -0.5$ (point A in Fig. 3), complete the specifications on $L_f(j\omega)$. In this case, $L_f(j\omega)$ is easily shaped to satisfy the $B(\omega)$ and $\omega_o = 4.5$, as shown in Fig. 3, with

$$L_f(s) = \frac{(1.28)10^6(s^2+30s+225)}{s(s^2+6.44s+21.2)(s^2+15s+225)^2}.$$

The prefilter G_3 is obtained from

$$G_3 = \frac{T_f(s)}{[L_f/(1+L_f)]} = \frac{1}{(1+10s+100s^2)(1+s)^2} .$$

Constraint (10) determines $G_2(s)$ by use of $X_e = \dfrac{Z_e}{N_f G_2 K P_h}$,

$N_f = \dfrac{2B}{\pi A}$ giving $|BG_2(j\omega)| \geq \dfrac{\pi \alpha\omega}{2K_{min}} \left|\dfrac{Z_e(j\omega)}{P_h(j\omega)}\right|$ for $\omega \leq \omega_o$,

with $|Z_e|$ the larger of $|Z_{er}|$, $|Z_{ed}|$, assuming simultaneous occurrence of extreme $r(t)$ and $d(t)$ is very rare. In this case, it is easily found that $|Z_{er}(j\omega)| > |Z_{ed}(j\omega)|$ for $\omega \leq \omega_o$, so $Z_e = R_e T_f$ is used. A satisfactory choice is

$$BG_2 = \frac{.76\pi(1+10s)(s^2+3.22s+21.2)}{(1+10s+100s^2)(1+s)^2} .$$

G_1 is found from $L_f = \frac{2B}{\pi A} G_1 G_2 K P_h$. B can be chosen for convenie in hardwareimplementation. The value $B = 16\pi$ was here chosen. $A_{min} = 0.5$ is known from knowledge of L_f, BG_2, G_1, KP_h.

Motivation for the Two-Loop SOAL

It is seen from the above that the presence of K_{max}/K_{min} in γ_1 of (9) is a very serious impediment in SOAS design, leading to $(\omega_o)_{min}$ unnecessarily large for the adaptive and disturbance attenuation needs of the system. Suppose K_{max}/K_{min} could be eliminated with $\gamma_2 = 2/m$ used instead of γ_1 in (9). Then, in the above example in Fig. 2, the intersection of $\gamma_2|Z_{re}(j\omega)|$ with $1/3\omega$ gives $(\omega_o)_{min} = 1.3$, in place of 4.5rps, an improvement of 1.79 octaves. Note that constraint (1b) is easily satisfied even at this value of $\hat{\omega}_o$. But in general there is some possibility that (1b) becomes significant, in the SOAL design. For $D_e = k/s$ or $k/(s+\alpha)^2$ etc., tolerable k_{max} is exactly $\frac{K_{max}}{K_{min}} = 100$ times the previous values. Thus, elimination of K_{max}/K_{min} in γ_1 is highly desirable and the SOAL (Fig. 4a) is a suitable structure for this purpose.

III. SOAL ADAPTIVE THEORY

In order to eliminate K_{max}/K_{min} from γ_1 in (9), it is necessary to eliminate K_{max} from (8) and K_{min} from Z_e in the discussion leading to (9). Both are due to K/A being invariant in (7b), which in turn gives the system zero sensitivity to uncertainty in K. Hence, K_{max}/K_{min} elimination with retention of zero K-sensitivity is possible, if instead of K/A, BK in (7b) is made invariant. But whereas A automatically tracks K such that in steady-state K/A is invariant, B can be made to do so only by means of a secondary loop. The function of the latter is to track A and adjust B, to force A to be constant in the steady-state. Since (7a) fixes ω_o and (7b) compels BK/A to be fixed (inasmuch as $G_1G_2\,P_h(j\omega_o)$ is fixed), fixed A compels BK to be invariant. This reasining leads directly to the self-oscillating adaptive loop (SOAL) structure of Fig. 4a.

SOAL Structure. In Fig. 4a, a peak detector A_{me} is used to measure the difference between the maximum and minimum values of $x = A \sin \omega_o t + x_f(t)$. This gives a fast and accurate measurement of 2A, because (recall 2a, b) the system is designed so that over the range of command and disturbance inputs, $|x_f(t)|$ is both small and slow relative to A and ω_o. (See later for comparison with other methods). The measured A_m is compared with the chosen reference value A_r and the difference $A_e(t)$, processed by $\psi(s)$, changes B so that in the steady-state, A_e is very small or zero. The dynamics of the secondary loop are next considered.

Dynamics of Secondary Loop

The technique [29] of the 'Fundamental Feedback Equation' is used to derive the dynamics. In a linear time invariant system, let $\mathcal{I}$ be any variable chosen as an independent system input, and $\mathcal{O}$ the output variable of interest. Let $\mathcal{R}(s)$ be the transfer function of any system element such that $\mathcal{S}(s) = \mathcal{L}(s)\,\mathcal{R}(s)$, where $\mathcal{S}$ and $\mathcal{L}$ are any two system variable so related to $\mathcal{R}$. If the element $\mathcal{R}$ is thus uniquely identified by the variables $\mathcal{S}$ and $\mathcal{L}$, then the relation between $\mathcal{I}$ and $\mathcal{O}$, has the form

$$T_{\mathcal{O}\mathcal{I}} \stackrel{\Delta}{=} \frac{\mathcal{O}(s)}{\mathcal{I}(s)} = t_{oi}(s) + \frac{\mathcal{R}(s)t_{ci}(s)t_{os}(s)}{1-\mathcal{R}(s)t_{cs}(s)} . \qquad (11)$$

The important feature of this 'fundamental feedback equation' for reference $\mathcal{R}(s)$, is that t_{oi} , t_{ci} , t_{os} , $t_{cs}(s)$ are completely independent of $\mathcal{R}(s)$ and they are calculated quite easily, as will be seen. Eq. (11) is therefore very useful if one wishes to focus attention on the relation between $T_{\mathcal{O}\mathcal{I}}$ and $\mathcal{R}$. It is also very useful in analysis [29] enabling a complex problem to be broken into four simpler ones-so used here.

Let the independent input $\mathcal{I}$ be a change in A and the output $\mathcal{O}$ be the change in B . Let $\mathcal{R}(s) = \psi(s)$, so $\mathcal{C}(s)$ and $\mathcal{S}(s)$ are as shown in Fg. 4a. The four transmissions $t_{ij}(s)$ are obtained as follows [29]. Cut the system at points a, a' with Fig. 4b replacing the top (secondary loop) part of Fig. 4a. The functions t_{oi} , t_{ci} are the transmissions from $\mathcal{I} = \Delta A$ to the variables $\mathcal{O} = \Delta B$, $\mathcal{C} = a$. The functions t_{os} , t_{cs} are the transmissions from $\mathcal{S} = a'$ regarded as an independent input, to the variables $\mathcal{O} = \Delta B$, $\mathcal{C} = a$. One must calculate four transfer functions instead of only one - but the cuts can make the latter four much easier to find - providing a wise choice of $\mathcal{R}_o$ has been made. The system input r(t) in Fig. 4b need not be zero so long as it may be assumed it does not affect the limit cycle.

Accordingly, in Fig. 4b

$$t_{oi} = \frac{\mathcal{O}}{\mathcal{I}}\bigg|_{\mathcal{S}=0} = 0 \quad ; \quad t_{ci} = \frac{\mathcal{C}}{\mathcal{I}}\bigg|_{\mathcal{S}=0} = -A_{me}(s)$$ being

the transfer function of a peak - to - peak detector, which requires a half - period, giving

$$t_{ci} = -e^{-.5sT} \quad , \quad T = 2\pi/\omega_o \quad ; \quad t_{os} = \left.\frac{\mathcal{O}}{\mathcal{S}}\right|_{\mathcal{I}=0} = 1 \; .$$

The function $t_{sc} = \left.\frac{\mathcal{C}}{\mathcal{S}}\right|_{\mathcal{I}=0}$ consists of two parts. Obviously one part is $-A_{me}(s)$. The other is the effect of ΔB on the limit cycle amplitude at x, precisely as in the SOAS. The steady-state value is obtained from (7b) giving $\frac{\Delta A}{\Delta B} = \frac{4}{\pi} |G_1 G_2 K P_h(j\omega_o)| = A/B$. The dynamics of the change ΔA due to ΔB was found experimentally to be well approximated by $\frac{L_f/\rho}{1+(L_f/\rho)}$ with ρ between 2 and 3 for different conditions. The worst case $\rho = 3$ is used here, giving the 'secondary loop transmission'

$$L_s(s) = -\psi t_{cs} = \frac{4\psi}{\pi}|G_1 G_2 K P_h(j\omega_o)| \; e^{-.5sT} \frac{L_f/3}{1+L_f/3} \qquad (12)$$

The stability and speed of this B adjusting loop is primarily determined by $L_s(s)$, whose maximum 'crossover frequency' ω_c (where $|L_s(j\omega_c)| \overset{\Delta}{=} 1$) is $< \omega_o/3$ approximately, because $e^{-.5sT}$ contributes $-60°$ phase at $\frac{\omega_o}{3}$ and allowing at least $30°$ phase margin, there is left $-90°$ needed for $|dL_s(j\omega)/d\omega| \approx -6$db per octave at ω_c. It is interesting to see the reappearance of the K uncertainty in L_s, after it was banished from the primary loop and quasilinearity conditions. The K uncertainty must be considered in the shaping of L_s. Since $\omega_{c\,max} \approx \omega_o/3$ at $K = K_{max}$, $\omega_{c\,min}$ at $K = K_{min}$

can be quite small with resulting much slower adjustment of B to K changes near $K = K_{min}$. The function $\psi(s)$ is available for shaping of $L_s(s)$.

From the above it is easy to find the effect of changes in K on B . The independent input $\mathcal{I}$ is now a change in K . Using the same $\mathcal{R}(s)$, the result is

$$\frac{\Delta B}{\Delta K} = -\frac{B}{K}\frac{L_s}{1+L_s} \quad ; \quad L_s \text{ of Eq. (12).}$$

<u>SOAL Design Equations</u>

These follow easily from those of the SOAS. In (8) BK is constant, so K_{max} is replaced by K . Constraint (9) becomes

$$\left|\frac{X_e(j\omega_o)}{A}\right| \geq \frac{2}{m} |Z_e(j\omega_o)| \tag{13}$$

because A is constant. For the same reason (10) becomes

$$\left|\frac{X_e}{A}(j\omega)\right| \leq \frac{1}{\alpha\omega} \quad , \quad \forall\ \omega \leq \omega_o \ . \tag{14}$$

The above is illustrated by a numerical example.

SOAL Design Example.

The plant and specifications are the same as in the SOAS example, except that m there of 1 is decreased here to .01. In this way,

$$\frac{2}{m_{new}} |Z_e(j\omega_o)| \text{ (SOAL)} = \frac{2}{m_{old}} \frac{K_{max}}{K_{min}} |Z_e(j\omega_o)| \text{ (SOAS)} \quad ,$$

giving again $\omega_{o\ min} = 4.5$ [If $m = .01$ is used in the SOAS, the resulting $\omega_{o\ min} = 14$ rps, and it is possible that over part of this range $|Z_{de}(j\omega)| > |Z_{re}(j\omega)|$, so even larger $\omega_{o\ min}$ would be necessary]. The bounds on $L_f(j\omega)$ to obtain the specified disturbance attenuation are the same as in the SOAS so precisely the same $L_f(s)$, $G_3(s)$, $BG_2(s)$, $G_1(s)$ result. In the SOAS, $A_{min} = .5\pi$, here fixed. If $B = 16\pi$ is used at $K_{min} = 1$, then $B = .16\pi$ at $K_{max} = 100$. Only $\psi(s)$ remain to be chosen. Choice of $\psi(s) = 0.4/s$ gives very simple secondary loop dynamics, because at $K = 100$, $A = .5$, $B = 16\pi$, $L_s \approx \frac{4}{s} e^{-.7s}$ (ignoring $(L_f/3)/[1+(L_f/3)]$ which is close to 1 over $[0,.4]$, .4rps being the crossover frequency ω_c of $L_s(j\omega)$. The time delay $e^{-.7s}$ gives 16° phase lag at ω_c . If this too is ignored,

$$\frac{\Delta B}{\Delta K} \approx \frac{.4}{s+.4} \quad .$$

<u>Design Verification.</u> The system was simulated on the CDC 6400 at University of Colorado, using MIMIC. Results are shown in Figs. 5a-c for abrupt gain changes in K. In (a), the time required for B to reach 95% of its final value is 9.8 seconds. If K = 100 is used in the previous simple first order approximation of L_s then three time constants plus a half-period predict 8.2 seconds, whereas use of K = 50 predicts 15.7 sec. It is seen that the speed of adjustment significantly decreases, as predicted, at the smaller K values. The invariance of the system step response for various K is seen in Figs. 6a-b; also that (2a) is very well satisfied.

<u>Comparison with other Two-Loop Self-Oscillating Adaptive Systems.</u>

One significant distinction of the SOAL is its retention of the nonlinear element N of the SOAS, with use of the second loop to modify its parameter. In some other two-loop systems [13, 18, 19, 21], N is eliminated and a gain changer is used in its place. Another crucial difference is the point at which A of $A \sin \omega_o t$ is measured. In the SOAL, it is measured at the point where it has the greatest immnunity to forced signal components, because the design guarantees that at x in Figs. 1, 4 the forced components are slow and small realtive to ω_o and A. Peak to peak detection becomes possible which further increases the immunity. In the others [18-22], A is measured closer to the plant input or output, where the forced components are much stronger. A bandpass filter becomes necessary with its additional significant time lag over and above that needed to measure A . Disturbance components can conceivably be misinterpreted as plant gain changes with possibly disastrous results. If the pass-band of the filter is narrowed to decrease the latter possibility, the greater is the danger if ω_o wanders out of the pass-band and the greater is the time lag of the filter.

General Plant Uncertainty.

In the above, only uncertainty in the plant gain factor K of $P = KP_h$ was considered, in order to emphasize the fundamental SOAS and SOAL properties. In the general uncertainty case L_f has adaptive obligations, which it does not have for K variations only and no disturbances. These obligations in a quantitative design can be put in the form of bounds on $L_f(j\omega)$, just like those in Fig. 3 for the disturbance attenuation obligations of L_f .

Reference [25] has treated in detail the general uncertainty problem for the SOAS and it was found that some iteration may be theoretically necessary, but rarely in practice because ω_o is relatively so large that $|L_f(j\omega)|$ is significant over a relatively large ω range. The treatment for the SOAL is precisely the same and therefore need not be repeated here. However, in the design execution, the iteration is much more likely to be necessary, especially in the case of large uncertainty in plant dynamics. The reason is that ω_o is much smaller in the SOAL than in the SOAS , so one does not have so much extra L_f bandwidth. The bounds on $L_f(j\omega)$ due to the uncertainty in P_h may be such that ω_o needed (at which $L_f = -0.5$) for this purpose, is larger than ω_o min obtained from constraints (13, 14). It is obviously easy to manufacture such problem specifications. In any case, the procedure is identical to that presented [25] for the SOAS.

IV. NONLINEAR SECONDARY LOOP SOANL

The logical next step for improving the adaptive system is to try to eliminate the sensitivity of the SOAL secondary loop L_s to the plant gain K . This can be done, in the steady-state, by means of a multiplier in the secondary loop whose multiplication factor is inversely proportional to K . Since K is not directly available, B which is inversely related to K is used, as shown in Fig. 7, which shows only the secondary loop, as the primary loop is identical to that in Figs. 1, 4a. The 'fundamental feedback equation' is again used to find the dynamics of the new L_s . As the loop is nonlinear, such a linear time invariant description, valid only for small perturbations about an operating point, is a function of the operating point. The easiest one is when the secondary loop is initially quiescent i.e. $A_r = A_m$ so there is no multiplier output.

Let $\mathcal{P}$ and $\mathcal{S}$ be as shown in Fig. 7. Let $\mathcal{I}$ be ΔK, a change in the plant gain, and $\mathcal{O} = \Delta B$. Make cuts at a, a'. Then $t_{oi} = 0$ due to the cuts and $t_{os} = 1$ To find $t_{ci} = \frac{a}{\Delta K}$, note that ΔA_m (recall similar calculation in the SOAL)

$$= \frac{4B(\Delta K)}{\pi} \left| G_2 P_h G_1(j\omega_o) \right| \; A_{me} \frac{L_f/3}{1+L_f/3} \, .$$

This is multiplied by $-B/B_{min}$, giving a result proportional to B^2. To find t_{sc}, let a' be an independent input and find a/a'. B becomes $B_{new} = B + a'$ giving

$$\Delta A_m = \frac{4a'}{\pi} \left| G_2 P_h G_1(j\omega_o) \right| \; K \, A_{me} \frac{L_f/3}{1+L_f/3} \, ,$$

which is multiplied by $-B_{new}/B_{min}$. For incremental a', the result is $t_{sc} = -\frac{-B}{B_{min}} \frac{\Delta A_m}{a'}$, and $L_s' = -\psi(s) t_{sc}$ is the SOANL secondary loop transmission. In the steady state the above calculated $\Delta B/\Delta K(s)$ gives $\Delta B/\Delta K = -B/K$, just as in the SOAL. The important difference is that L_s' contains BK (cf K in (12) for the SOAL), which is invariant in the steady-state. Hence, the SOANL should deliver for all $K \in [K_{min}, K_{max}]$, the fast adjustment to K variations, which the SOAL can do only near K_{max}.

SOANL Design Example

The synthesis procedure, specifications etc are the same as for the SOAL. Hence, the results for G_1, G_2, B etc are precisely the same. Simulation results are shown in Fig. 8. The analysis made was for small K changes, and would be expected to be valid for large changes if these were somewhat slower than the secondary loop dynamics. Nevertheless, the same basisc results hold even for very large and abrupt changes in the plant gain K, as seen in Fig. 8.

V. CONCLUSIONS

The development of a quantitative design theory for the single loop self-oscillating adaptive system (SOAS) has led in a natural, logical manner to the two-loop system (SOAL) with important superior features. The emphasis on quantitative design reveals also the superiority of the SOAL over other such two-loop structures. It is interesting that the gain factor uncertainty K_{max}/K_{min} which was inherently banished as an adaptive problem by the oscillation, returns in the SOAS in the quasilinearity constraints. It plays there an important role in forcing an unnecessarily large $\omega_{o\ min}$. It is banished in the SOAL, permitting a much smaller $\omega_{o\ min}$, but reappears in the secondary loop dynamics. This results in the speed of adjustment to K variations being a function of K and much slower at K_{min} than at K_{max}. It is banished from the SOAL by means of nonlinear compensation in the secondary loop, giving the SOANL. It undoubtedly reappears in a subtle manner in a nonlinear analysis of the SOANL , but if so, it does not appear to be a critical one. Exactly the same two-loop extensions can be made to the externally excited adaptive (EEAS) system [26], giving multiple loop EEAL and EEANL which therefore do not require separate treatment.

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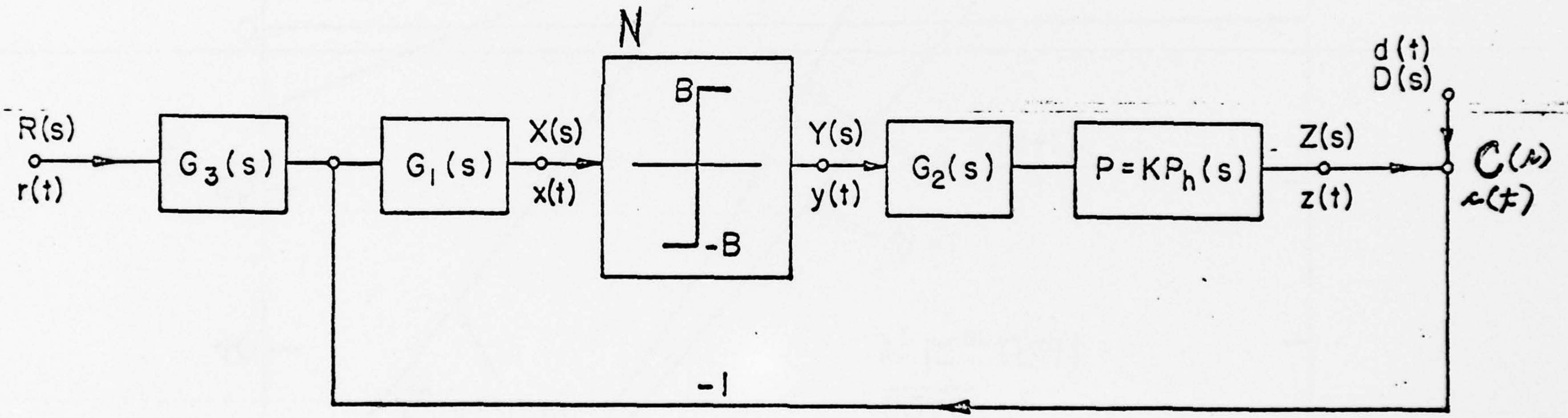


Fig.1 Single loop self-oscillating system-- SOAS

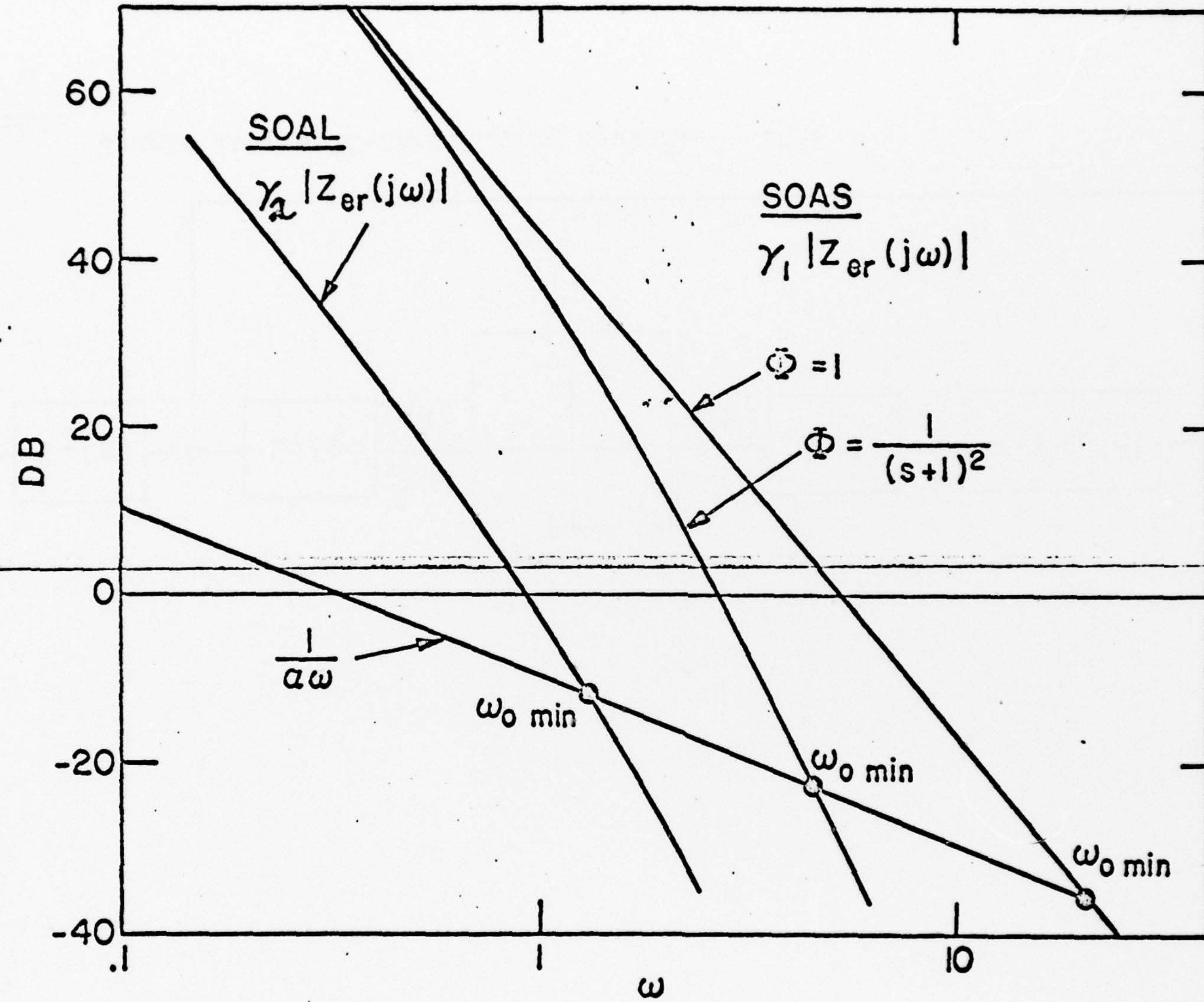


Fig. 2. Design constraints on Bode plot

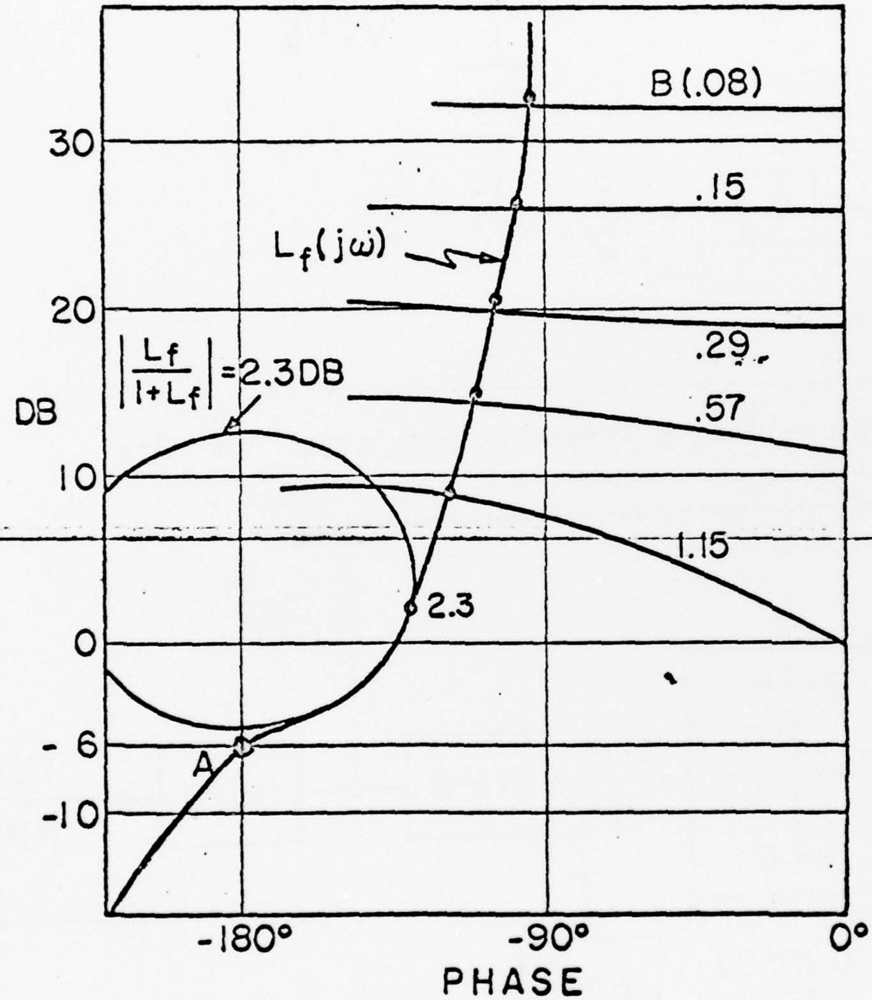


Fig.3. Bounds $B'(\omega)$ on $Lf(j\omega)$ for disturbance attenuation

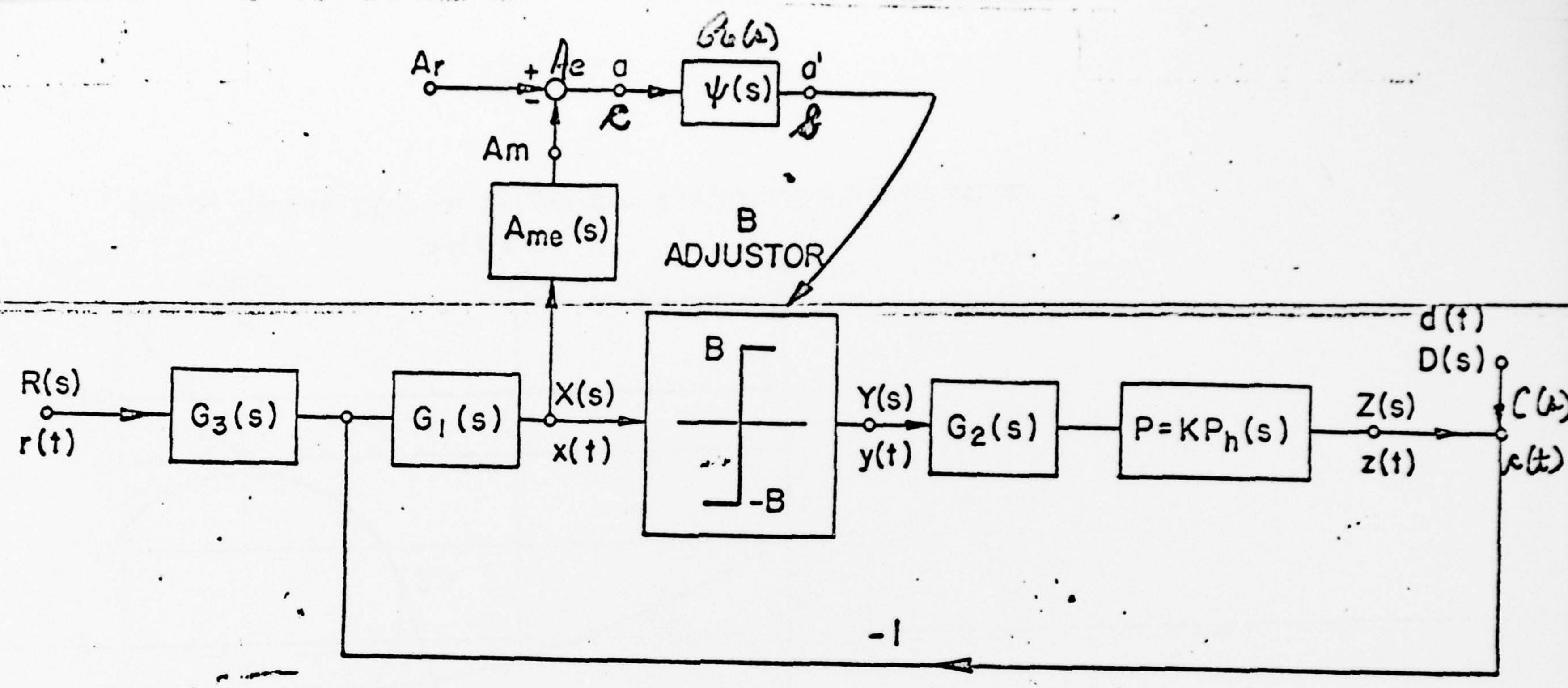


Fig. 4a. SOAS with adjusting B loop -- SOAL

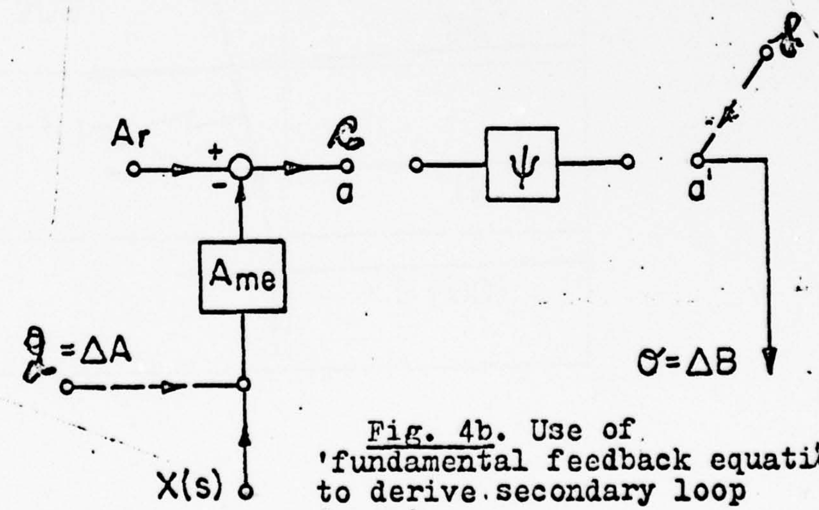


Fig. 4b. Use of 'fundamental feedback equation' to derive secondary loop dynamics

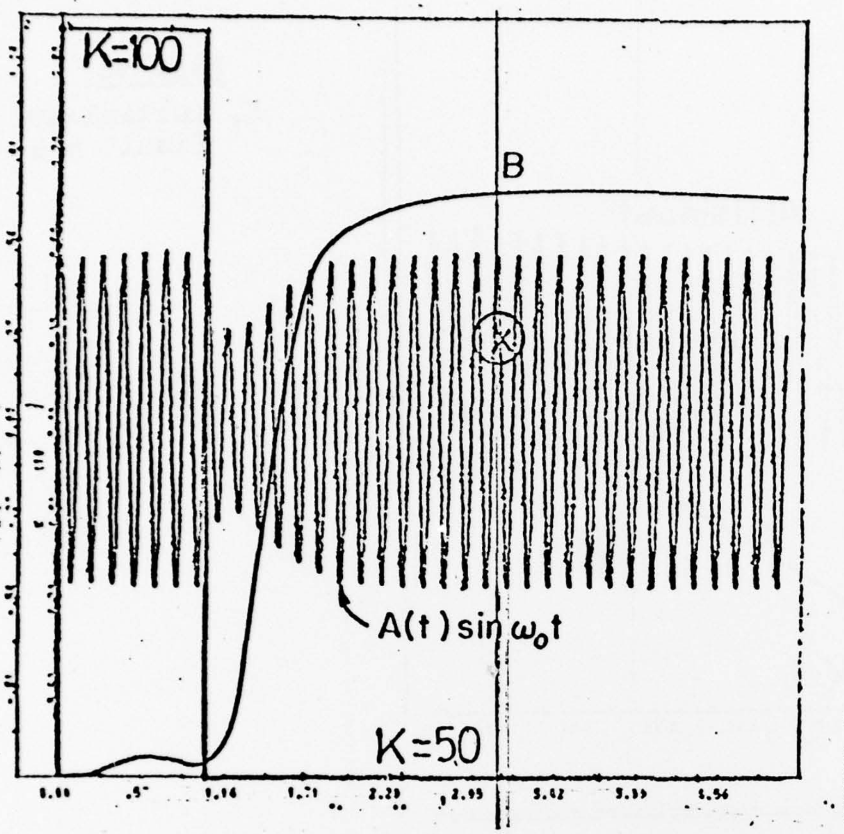
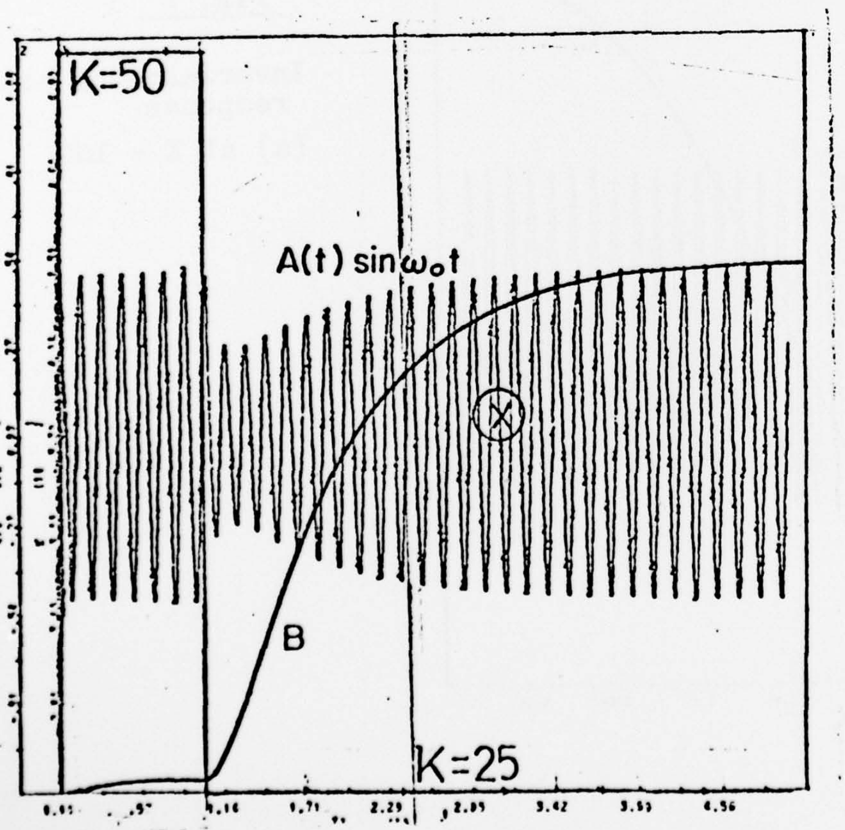


Fig. 5. SOAL Design Example.

Effect of abrupt K changes on B(t) and $x_o(t) = A(t) \sin \omega_o t$.

(a) K initial = 100
final = 50



(b) K, initial = 50
final = 25

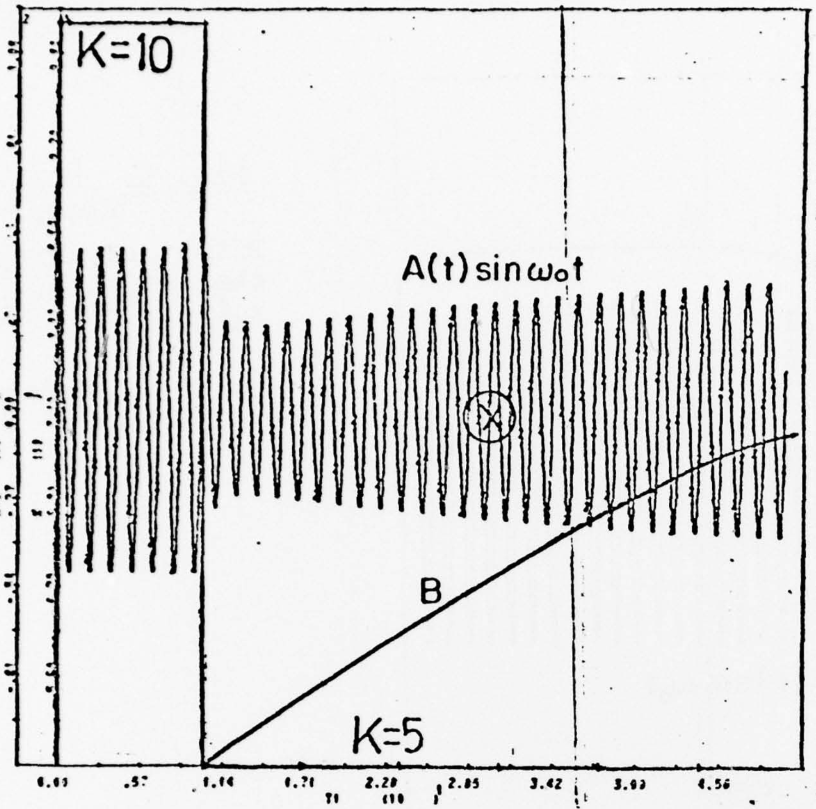


Fig. 5c

K, initial=10
final = 5

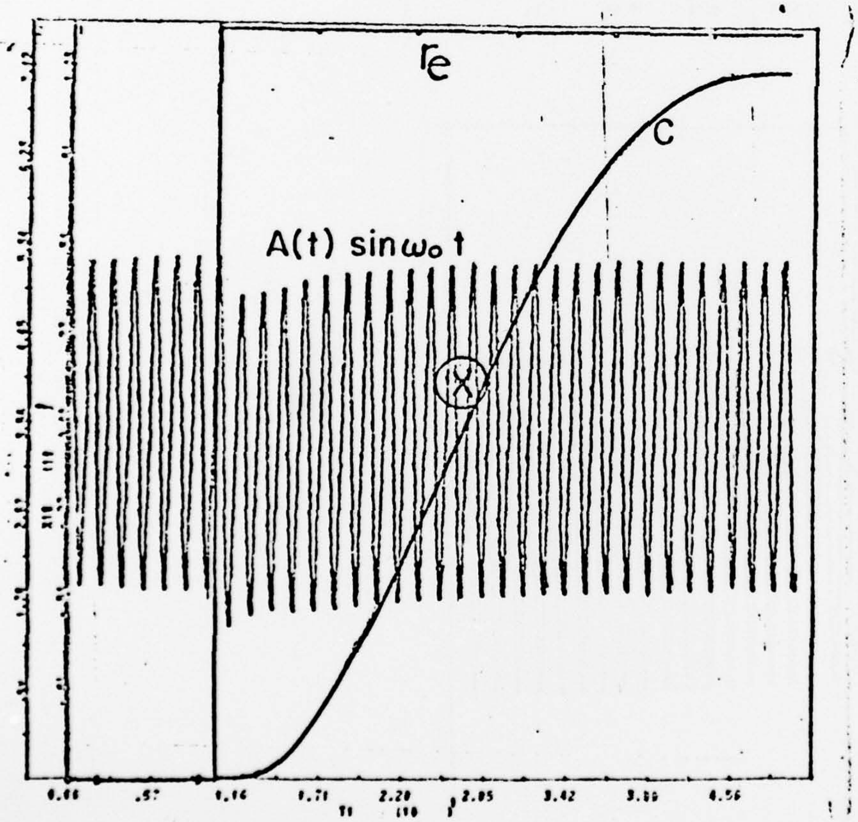
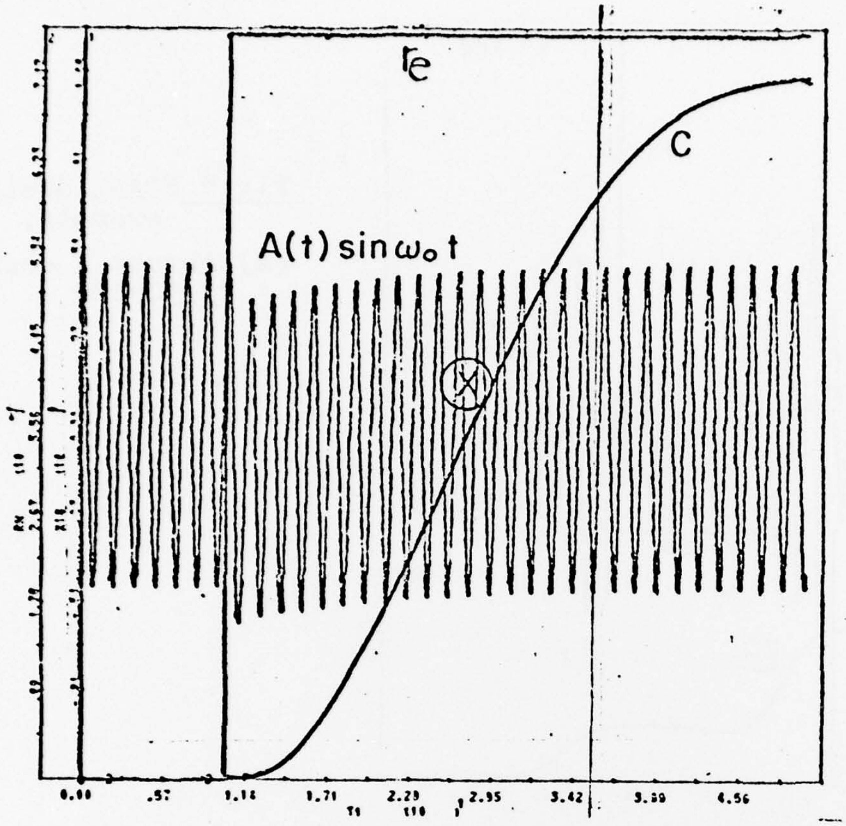


Fig. 6

Invariance of step response

(a) at K = 100



Fig, 6b

at K =1

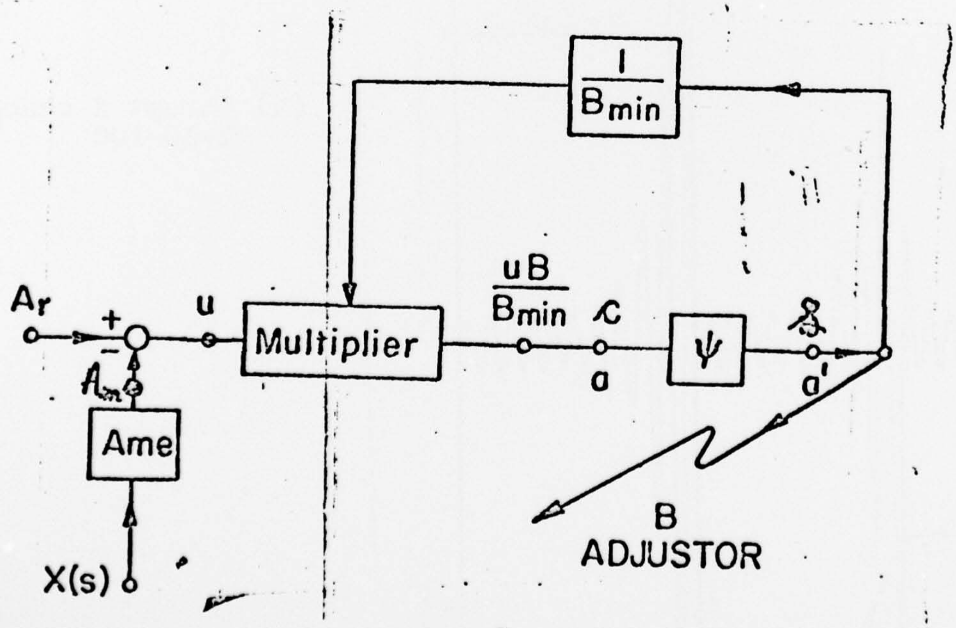


Fig.7

Three loop system (SOANL) for eliminating K_{max}/K_{min} from secondary loop dynamics

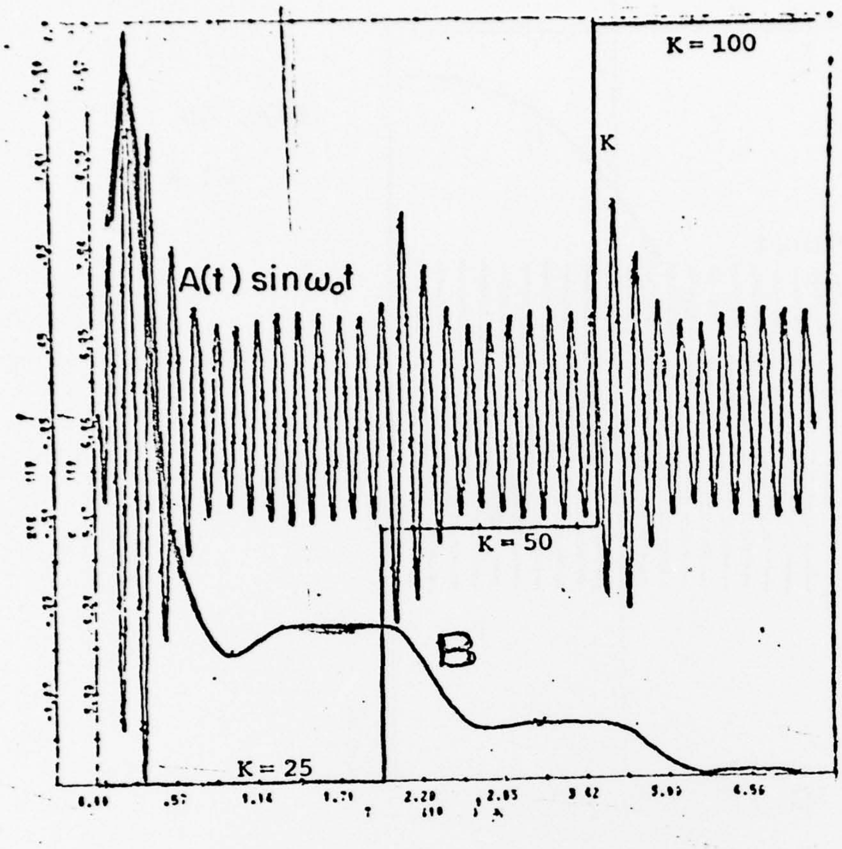
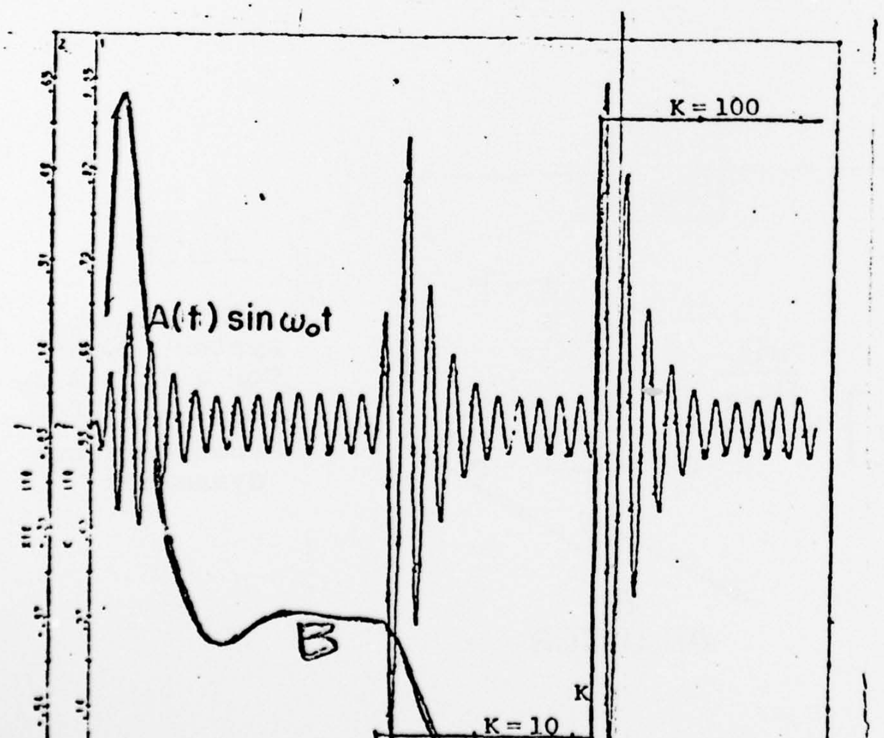


Fig.8 SOANL design example.

(a) Abrupt K changes 25-50-100



(b) Abrupt K changes 1-10-100

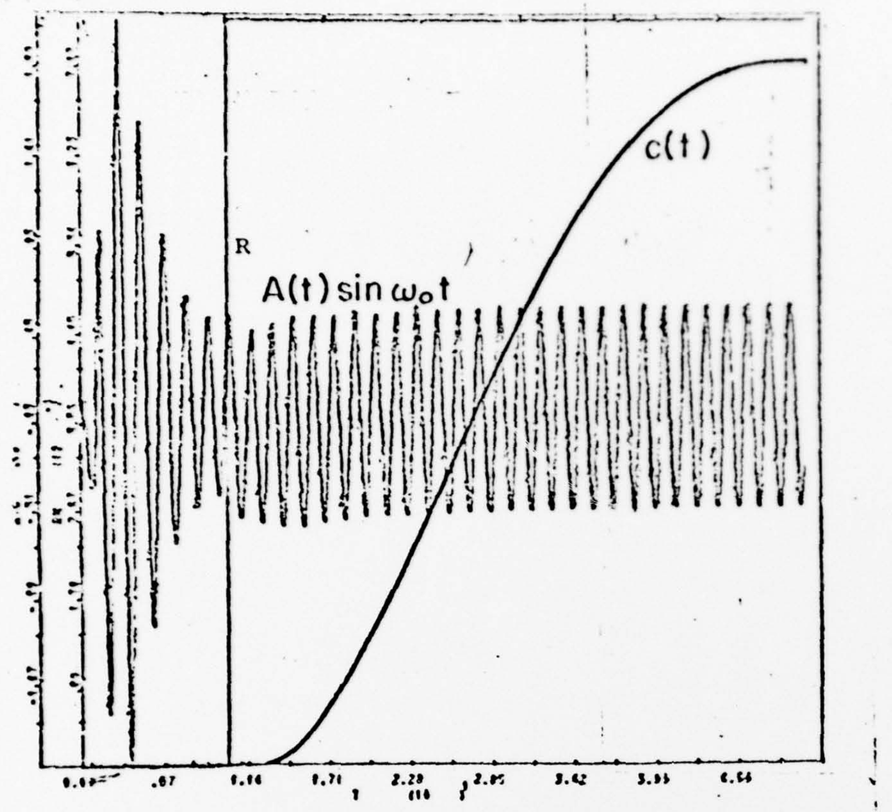


Fig. 8c.

SOANL design example
Step response at K=100

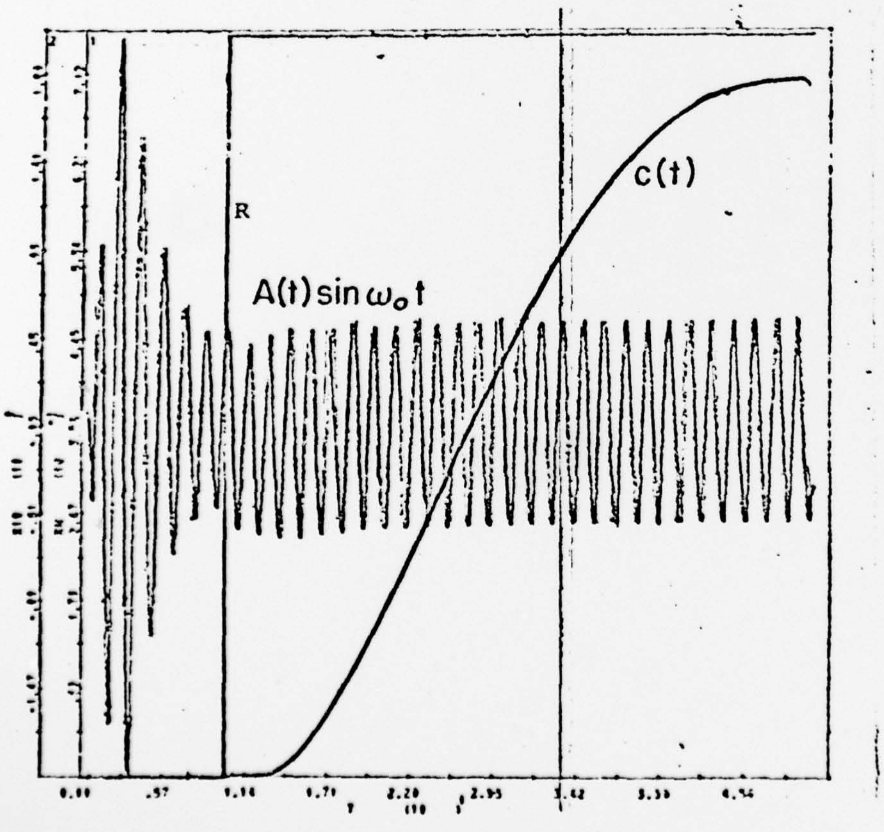


Fig. 8d

Step response at K=1